Introduction

Welcome to the first PROMYS Math Circle problem set of the 2018-19 school year! If you've done other PMC sets, you'll already be familiar with the following guidelines. But if this is your first time working on PMC sets, a very special welcome to you, and here are some things to keep in mind.

- Our guiding principles are **Exploration**, **Explanation**, and **Fun**.
- **Exploration:** Don't worry about finishing all the problems, or even finishing a particular problem. If you get partway through a problem, and find your own questions to ask, that's great! Go for it and answer your own questions. Those are the best kind.
- **Explanation:** Getting to an answer is only valuable if you are able to communicate your answer to someone else and explain how you got there. Explaining mathematics clearly and precisely is not easy (just ask your math teacher!), but it is important to practice this important skill! Even when something seems "obvious" (especially when something seems obvious, perhaps), make sure you can explain it so others can understand.
- **Fun:** This one is self-explanatory. Fun, have it!

Some Probabilities

P1. Find the probability that when you pick two numbers between 1 and 5 (inclusive, with the possibility of picking the same number twice) that they do not share a common factor greater than 1. Repeat for picking numbers between 1 and 6, 1 and 7, 1 and 8. There are multiple ways to do this problem. How many can you come up with?

P2. What is the probability that two positive integers don't share a common factor greater than 1 if

(i) one of the numbers is 1?

(ii) one of the numbers is 2?

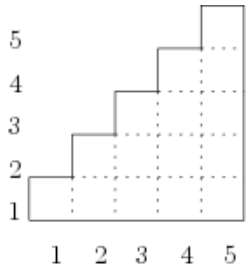
(iii) one of the numbers is 3?

(iv) one of the numbers is 6?

(v) one of the numbers is 9?

P3. What is the probability that two positive integers aren't both divisible by 2? Aren't both divisible by 3? Aren't both divisible by some prime p?

P4. Consider the "staircase" with 5 steps drawn below.



Add a dot to the (m, n) box if m and n are relatively prime, i.e. share no common factor greater than 1. For example, you would add a dot to the 3rd column, 2nd row because 3 and 2 are relatively prime. How many dots are there?

P5. Make staircase pictures of other sizes. If you have the staircase picture with 100 steps, there are 3044 dots. Can you use this to say the number of dots in the staircase with 101 steps?

P6. How can you use the staircase pictures to answer the questions from P1?

P7. Make a chart with the following information: steps in the staircase, number of dots, staircase area (total number of squares in the staircase), and proportion of squares with dots. What do you notice as the number of steps gets larger?