PROMYS for TEACHERS

Learning in the Spirit of Exploration

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PROMYS for TEACHERS is a collaboration of Boston University's Department of Mathematics with the Education Development Center in Newton, Massachusetts. Together we are working to promote what we call a "culture of exploration" in high school mathematics classrooms. Our experience proves that the joys of exploration and discovery can be experienced by high school students and teachers in ways that are not all that different from what a seasoned mathematics researcher experiences. Moreover, teachers who have such experiences in their backgrounds are better prepared to encourage independent inquiry among their own students. Reports from former PROMYS teachers confirm that *all* students benefit from this kind of instruction, not just the most talented ones.

History of PROMYS.

The Program in Mathematics for Young Scientists (PROMYS) has existed for over a decade at Boston University as a program that engages mathematically inclined high school students in the process of mathematical exploration through their work on unusually challenging problems in number theory. Since 1991, PROMYS has also worked with pre-service high school mathematics teachers from the Boston University School of Education. Beginning in the summer of 1999, PROMYS expanded its activities again by engaging in-service Massachusetts high school mathematics teachers in the program's summer activities and running five professional development seminars at the Education Development Center (EDC) during the academic year.

PROMYS has its roots in the famous Ross Young Scholars Program, which is still running strongly at the Ohio State University in Columbus forty four years after its beginnings at Notre Dame in 1957. Many of the PROMYS faculty are alumni of the Ross Program and much of our inspiration comes from the experiences we shared in the late sixties and early seventies as young participants in Arnold Ross's program.

Elements of the Program.

PROMYS for TEACHERS is built around three major components - (1) immersion in an experience of mathematical exploration for six weeks in the first summer; (2) reflection on the relevance of this experience for classroom practice during the intervening academic year; and (3) more immersion in mathematics in the second summer. Teacher participants receive a stipend of $1,800 for each summer of participation in PROMYS. In addition they receive 8 graduate credits in mathematics for their work in the summer and an additional 4 graduate credits in mathematics education for their work in the academic year workshops.

The PROMYS Community.

The PROMYS community is divided into several closely interacting groups who bring a variety of experiences at a variety of levels to the program. The first-year participants consist of10 in-service teachers, 5 pre-service teachers, and 45 high school students. Another 10 teachers attend the program as returning participants along with 15 returning high school students. Teachers

commute from home each day, but the students, who are recruited nationally, live in the dormitories. A staff of 8 counselors works with the teachers on problem solving activities during the day and reviews their written work each evening to be returned with comments the next morning. The teacher counselor staff is made up of former PROMYS teacher participants and graduate students from the department of mathematics. An additional staff of 15 undergraduate counselors lives and works with the high school students in the dormitories. The student counselors are recruited from around the country. Many are former participants in the program. The PROMYS faculty has 7 members, two of whom come from the Education Development Center while the other five are university mathematicians, many of whom were participants in the Ross Program in their high school years. The faculty work together with participants on problem solving and in exploration labs.

The significance of the depth of this community cannot be overemphasized. PROMYS aims to change participants' attitudes about mathematics. The community is by far the most effective tool for bringing about this change. Most participants find their experience in PROMYS to be overwhelming at first. They must work hard to adjust to the demands of PROMYS and its unusual philosophy of learning. Beginning participants work together in groups and often discuss their ideas with returning participants and counselors. Returning participants share their valuable insights and offer encouragement to the beginning participants. Assuring that the participants find the challenges of PROMYS to be exciting and fulfilling is probably the most important function of the counselors. The role models provided by returning participants and counselors, and the personal and accessible feedback they offer, are indispensable features of the PROMYS experience.

The Summer Program.

The PROMYS summer activities combine three tightly interlocking components: (1) problem-solving activities shared by all first-year participants; (2) independent team projects carried out by all participants in groups of four; (3) advanced activities (seminars, and projects) for returning participants. These components are supported by discussion meetings and independent problem-solving activities with the counselor staff.

(1). Problem Solving Activities. All first-year participants engage in intense problem solving in number theory designed to introduce them to the methodology of mathematical research. We use the magnificent collection of problems designed by Professor Arnold Ross for use in the Ross Program. These problem sets provide participants with valuable insights into the research process. Particularly effective is the way the problem sets build on themselves over the course of the program. Simple numerical observations made in the early problem sets are enriched and extended in later sets so that participants are treated to a first-hand experience of unraveling deep and significant truths out of simple ideas. Lab projects (described below) build on particular threads that weave through the problem sets and introduce participants to some of the many unsolved problems that still remain. By engaging all participants in shared problem-solving activities, PROMYS creates a strong community of closely interacting teams working on independent but related projects, thus recreating the crucial element of communication and sharing of ideas that is so important to all research mathematicians.

(2). Team Activities. Every participant is assigned to a team of four participants who will work together, in consultation with the PROMYS faculty and staff, on an independent open-ended project. These projects extend particular facets of the general problem-solving activities described

above. Each team consists of one teacher and three students, sometimes including a student from the teacher's high school. The topic for the project is selected from a list of proposals prepared by the PROMYS faculty. Each project contains parts that are immediately accessible to all participants, and then continue with open-ended explorations, including unsolved problems. The summer 2001 projects for first-year participants include explorations of the postage stamp problem, generating functions, the calculus of finite differences and sums, partitions, the game of SET, repeating decimals in various bases, congruence properties of Pascal's Triangle, Pick's formula and Euler's Theorem on simple graphs, Egyptian fractions, and sums of squares.

Each team member keeps a notebook for recording progress as it is made, as well as new ideas, conjectures, and questions that arise along the way. The results of numerical experiments are shared and preserved in these notebooks. At the beginning of the third week, team progress reports are reviewed the PROMYS counselors and faculty and returned with feedback and suggestions. In the fourth week, the results of each project are written up carefully in the form of a mathematical research paper and submitted to the PROMYS faculty and in the fifth week each team jointly gives an oral present of their work to the rest of the program.

(3). Activities for Returning Participants. Returning participants take part in advanced seminars designed especially for them on topics that vary from summer to summer (e.g. discrete mathematics, geometry and topology, and the theory of equations). The PROMYS 2001 offerings for returning participants are: Combinatorics; Random Walks on Groups; and Analysis of Algorithms. Like the first-year participants, the advanced participants are also divided into teams of four, including one advanced teacher participant, and these advanced teams work together on projects designed to connect quickly with accessible research projects, the results of which may be publishable in research journals. The advanced projects in 2001 include investigations of arithmetic functions in number fields, generalizations of perfect numbers, change-making problems in combinatorics, and decompositions of the circle.

Returning participants add an important dimension to PROMYS by sharing their valuable program experience and by serving as peer role models for first-time participants. The second summer of participation helps teachers cement their experience of mathematical exploration and extend their content knowledge into other mathematical fields relevant to the high school curriculum. Moreover, through their discussions with beginning participants, the returning teachers share strategies they have learned for engaging students in the process of mathematical exploration. This is an important part of the community-building that is so crucial for the long-term success these efforts.

Academic Year Pedagogy Workshops

The workshops are designed and delivered by Al Cuoco and Michelle Manes of the Education Development Center (EDC) with support from Boston University's Department of Mathematics and School of Education. Teachers meet with mathematics education researchers, mathematicians, and other teachers to discuss curriculum issues, design projects, and to develop teaching methods that will engage the broadest possible spectrum of high school students in meaningful mathematical exploration. The seminars help teachers develop age-appropriate research experiences for their entire spectrum of students and to ``unpack'' the pedagogical approaches used in PROMYS to enrich the school curricula. Another important goal of the workshops is to establish an ongoing network (including an electronic network) of teachers, mathematics educators, and research mathematicians.

Each workshop begins with a concrete mathematical activity for the teachers that is easily accessible to students. Then deeper aspects of the activity are examined, in some cases leading right into contemporary research articles. Past topics have included cryptography, complex numbers and chaos, Pythagorean and Eisenstein triples, and areas (and volumes!) through paper-cutting and folding. Each workshop also provides time in which individual teachers can present samples of their own classroom activities and to discuss actual situations that have arisen in their classrooms. In the afternoons, teachers may design curriculum units for use in their classrooms.

Drawing on years of experience designing excellent curricula for the high schools, Al and Michelle are able to give the teachers valuable insights into the preparation of classroom activities centered in exploration. At one workshop, the participants were treated to a view of five or six different textbooks in geometry, all in current use. The discussion centered on an analysis of each, in order to understand how each was designed to accomplish its own distinctive goals. Another workshop was devoted to ways in which Geometer's Sketchpad can be used to stimulate student explorations. Other workshops explored Computer Algebra Systems and using the internet as a research tool.

One of the most gratifying features of the workshops has been the ongoing participation of former PROMYS participants. PROMYS teachers are expected to participate in only one year of the academic year workshops, but in fact almost every teacher from the first year attended at least one workshop in the second year as well, and many attended several. This attests to the high value that teachers place on the EDC workshops and is also an indication of the coherence the teachers feel as a group. The closeness of this mathematically-centered community is undoubtedly the most valuable asset of PROMYS for Teachers.

PROMYS Goals and Philosophy: A Culture of Exploration.

PROMYS for TEACHERS is designed to support current efforts nationwide to *change the culture* in high school mathematics classrooms to one centered on exploration and problem solving. The program fosters a *community* of teachers, young people, research mathematicians and educators all working together and sharing intellectual challenges. The program activities are designed to encourage the kinds of *habits of thought* that every researcher (not just of mathematics) uses in his or her efforts to understand the world around us. This combination of community and rich intellectual environment is a potent tool for engaging participants at a very personal level in an experience of exploring significant ideas.

Exploration is at the heart of mathematics. Mathematics is not just a powerful tool for understanding the world around us; it is also a powerful tool for discovering the world around us. When students learn mathematics through exploration, they experience a side of our subject that lies much deeper than the simple skills of algebraic manipulation and calculation. There are many opportunities in the high school curriculum for students to experience mathematical discovery. Teachers need the preparation and the confidence to recognize and take advantage of these opportunities. Here are some reasons for including a strong component of immersion and exploration in the professional development of teachers.

- *To emphasize the human side of mathematics.* Intellectual exploration is unique to the human experience. For students, mathematics often appears to have a rigid and hostile quality, which feels very different from the spirit of other human activities. By exclusively emphasizing calculation and symbol manipulation in the classroom we reinforce these misconceptions. Exploration, on the other hand, can be done in many ways. Teachers find numerous

opportunities in PROMYS to try out some of these ways for themselves. The adventures they experience come complete with the frustration of dead ends, the excitement of finding a promising new trail, the sense of triumph at surmounting a hard problem, and the sense of wonder at the beauty of the unexpected mathematical vistas that come into view. Teachers who have had such an experience of learning in the spirit of exploration are better prepared to bring this spirit to their classrooms. One PROMYS teacher told us:

> *… the passion for and the approach towards mathematics can only be learned by being immersed in mathematics.*

Another teacher wrote:

> *PROMYS has had profound effect on my teaching, mostly by reminding me why I love mathematics in the first place---the joy of discovery.*

- *To encourage student independence.* Learning is an intensely personal experience. No two students understand mathematics in the same way. Students rarely learn by listening to teachers lecture in the classroom. They learn from experiences that engage them personally and at their own level. By emphasizing exploration in the classroom, teachers can foster a climate in which such experiences occur frequently. One teacher described this as follows:

  > *I'm now having great success with students of all ability levels. Posing the proper questions allows my brighter students to dive deeper into topics than their classmates. More importantly, my slower students feel good about making progress at their own pace, resulting in increased confidence (perhaps the best motivator).*

- *To prepare students for life in a rapidly changing world.* The importance of flexible learning skills is especially apparent in these times of rapid technological progess. To keep up with international economic competition, it is increasingly apparent that the American workforce needs learning skills that adjust easily to changing demands. An experience of immersion in mathematical exploration can give teachers added confidence in their own abilities, so that they can recognize and appreciate unorthodox student approaches to problems and respond with the kind of flexibility that encourages independent student thinking. Here is how one teacher describes this process:

  > *By viewing my role as a "guide to student learning" rather than an instructor, I can remain focused on asking the right leading questions and resisting the temptation to show them what I know.*

- *To develop habits of thought that favor hard problems.* It is all too common that students tremble with trepidation in the face of hard problems. By emphasizing the process of discovering mathematics along with the results of those discoveries we can foster a student culture which relishes the prospect of exploring the unknown. Of course, this also requires readiness on our part to reward students for unsuccessful attempts.

> *I find that as a teacher it is sometimes easy to forget that our students don't always understand everything we teach …. One thing that PROMYS helped me to do is to help students to learn to appreciate being challenged and frustrated because that means they are thinking about the problem. PROMYS helped me to become better at asking questions rather than just giving answers.*

- *To encourage sharing of ideas.* Everyone enjoys talking about interesting discoveries they make. Such interaction can be organized formally in the classroom by asking students to work in small groups, or by asking students to describe their ideas to the class. Best of all, however, is when students independently talk to each other outside of the classroom. By offering encouragement and support to teachers as they use open-ended problem-solving techniques in their classrooms, PROMYS hopes to contribute to a changing culture in the schools that will inspire many interactions of this kind.
- *Exploration is what mathematicians really do.* Learning through exploration provides students with realistic opportunities to experience mathematics the way a professional scientist does. It is impossible to understand the process of mathematical research without having experienced it for oneself at some level. Exploration in the classroom provides opportunities to discuss the dynamic nature of modern mathematics in a context that simulates actual experiences of mathematical discovery. To lead these explorations effectively, teachers must feel and think like mathematicians themselves. One PROMYS teacher summed up his approach to teaching as follows:

> *These are not things that I do consciously. Rather, they're merely reflections of how I have learned to approach mathematics. Thus, I could've only learned how to teach the way I teach by studying mathematics intensely on my own.*